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SQUIDS-SNAPPER FISH DYNAMICS MODEL WITH FISHING EFFECTS IN TERENGGANU, MALAYSIA

MOHD ROSLAN, U.A.[1,2*], MOHD SAFUAN, H.[3] and MAT PIAH, R.[4]

[1]*School of Informatics and Applied Mathematics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia*
[2]*Marine Management Science Research Group, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia*
[3]*Department of Mathematics and Statistics, Faculty of Science, Technology and Human Development, Universiti Tun Hussein Onn Malaysia, 86400 Parit Raja, Johor, Malaysia*
[4]*School of Fisheries and Aquaculture Sciences, Universiti Malaysia Terengganu, Malaysia*
[*]*E-mail: ummuatiqah@umt.edu.my*

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ABSTRACT

Overexploitation of marine resources by human activities has become a pandemic issue nowadays. High fishing rates for example, may lead to the extinction of marine populations. In this paper, we introduce a mathematical model of prey-predator system for marine ecosystem with fishing rates for the case of Terengganu state. For this model, we use squids as prey while snapper fish as predator. The objectives of this paper are to analyze the sustainability of equilibrium populations of squids and snapper fish using stability analysis and to show the effect of fishing rates on both of these populations. This model shows that there are four potential equilibria solutions where both populations of squids and snapper fish may be extinct, mutual exclusions where either one of the species dies out as well as coexistence of both populations. The results for stability analysis reported that the equilibrium of coexistence of both populations was stable while the other was unstable. This means that populations of squids and snapper fish are estimated to sustain in the future with the current fishing activities in Terengganu. Hence, we conjectured that in order to guarantee both populations continue to exist, the fishing activities in Terengganu must be restricted within certain range of parameters that is lower than the population growth rates.

Key words: prey-predator model, stability, marine ecosystem, squids-snapper fish, Terengganu

INTRODUCTION

Overexploitation of marine ecosystems by human activities has been recognised as a global issue by the public worldwide including Malaysia. High fishing rates for example, has been evident as one of the determinants to the extinction of marine populations. This matter leads to the question of how well the systems can be managed in order to sustain the marine populations from extinction. To answer this question, prey-predator dynamics model is deemed as one of the prominent tools to investigate the dynamics processes of the systems.

The earliest mathematical model describing interacting population between prey and predator was proposed by Alfred Lotka in 1925 and Vito Volterra in 1926. Their model of prey-predator system is called as a Lotka-Volterra (Volterra, 1926). The equation of Lotka-Volterra model is given by

$$\begin{aligned} \frac{dx}{dt} &= \dot{x} = rx - \alpha xy, \\ \frac{dy}{dt} &= \dot{y} = \alpha xy - dy, \end{aligned} \tag{1}$$

where r, a, d and t represent the prey growth rate, predator attack rate, predator death rate and time respectively. The notations of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ denote the rates of change of prey and predator populations respectively over time.

There were reasonably numerous dynamics models have been developed to investigate the interaction between prey and predator in marine ecosystems. Most of them were associated with fisheries. Spencer and Collie (1995) proposed a

* To whom correspondence should be addressed.

model on exploited marine fish population where the growth rate of predator depends on predation and alternative prey. Moreover, Das *et al.* (2009) investigated the effect of toxic substance on the prey-predator fishery. Essentially, the idea of incorporating the effect of toxins on prey and predator was enthused from Smith (1974). Also, Lv *et al.* (2013) investigated a prey-predator model, which consists of two preys and a predator, where one prey fish species is placed inside an unreserved area while the other in a reserved area. In the reserved area, predation and fishing are prohibited. These two types of areas have also been considered by Biswas *et al.* (2017) where they focused on the dynamics of fish production such as the existence of equilibrium points as well as the conditions of stability and instability of their proposed model. A recent study by Keong *et al.* (2018a) has found the effects of toxicity on both prey and predator species in fishery model.

In this paper, our main objective is to develop a marine prey-predator model for squids and snapper fish with fishing effects. Consequently, we will analyze the stability of both populations for the chosen values of parameters as well as presenting the effect of fishing rates on both populations by varying a parameter in the model. We will apply this model to real data of fish landings in Terengganu obtained from the Malaysia Department of Fisheries.

A mathematical model for squids-snapper fish will be developed by incorporating two types of parameters. The first includes life-history characteristics such as prey growth rate r, predator growth rate s, predator attack rate a and carrying capacity K. The second includes management criteria such as fishing rates on prey and predator (denoted by E and H respectively). Let x and y denote the squids and snapper fish respectively. A flow diagram for our proposed model is depicted in Figure 1.

From the Figure 1, a nonlinear differential equation for squids-snapper fish with fishing rates can be constructed as:

$$\begin{aligned} \frac{dx}{dt} &= \dot{x} = rx\left(1-\frac{x+y}{K}\right) - \alpha xy - Ex^3, \\ \frac{dy}{dt} &= \dot{y} = sy\left(1-\frac{x+y}{K}\right) + \alpha xy - Hy^2, \end{aligned} \tag{2}$$

where all symbols in model (2) are described as in Table 1. Note that this model is in fact the modification of Lotka-Volterra model in (1). In model (2), the term $rx\left(1-\frac{x+y}{K}\right)$ represents logistic growth rate for squids with carrying capacity K, while the snapper fish growth rate is given by $sy\left(1-\frac{x+y}{K}\right)$ with the same carrying capacity K. Notice also that for both squids and snapper fish equations, we have the same term of $\left(\frac{x+y}{K}\right)$. This indicates that both species are sharing the same carrying capacity K. Some examples of carrying capacity are water, nutrient, oxygen and living space. The term αxy shows the interaction between squids and snapper fish with snapper fish attack rate α. In this situation, the squids suffer while the snapper fish gains. We call E and H as the fishing rates for squids and snapper fish populations respectively. Here, we assume that the catching term for squids is Ex^3 while the catching term for snapper fish is Hy^2. These mean that when the fishing activity operates, the number of squids being caught is greater than the number of snapper fish. The same approach has been done by Das *et al.* (2009) where they use the cubic term for the prey and quadratic term for the predator,

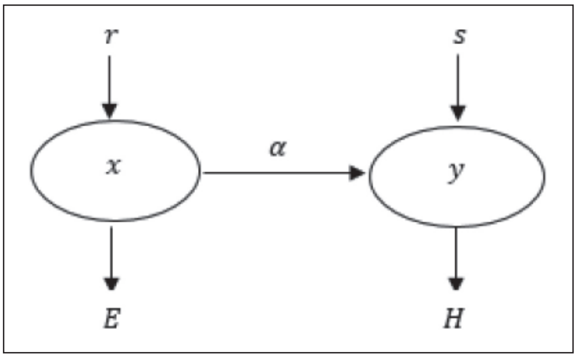

Fig. 1. Flow diagram of the model (2).

Table 1. Description and values for variables and parameters for squids-snapper fish model in (2) with fishing rates values obtained from Malaysia Department of Fisheries in Terengganu

Variables/Parameter	Description	Values
x	Number of squids population	unknown
y	Number of snapper fish population	unknown
t	Time (years)	[0, 50]
r	Squids growth rate	6.5
s	Snapper fish growth rate	0.5
K	Carrying capacity shared by both squids and snapper fish	100
α	Snapper fish attack rate	0.006
E	Fishing rate for squids	0.05
H	Fishing rate for snapper fish	0.05

except that they consider for the case of toxicity effect on both prey and predator. In Table 1, the values of parameters r,s,K,α are assumed while E and H are computed from the real data obtained from Malaysia Department of Fisheries in Terengganu. Note also that we assume both values of fishing rates on squids and snapper fish are same since they are caught together by the fishermen. All parameters in model (2) are assumed to be positive.

MATERIALS AND METHODS

In this section, we discuss two methods that are used to analyse model (2): stability analysis and modification of values of fishing rate parameter.

Stability analysis

Here, we discuss a method called the *stability analysis*. This analysis is valuable to determine the stability of equilibrium solutions of the dynamical systems. In our model, this analysis will be used to investigate the stability and instability of prey and predator populations over time. Lynch (2014) suggests five steps to implement this method.

Step 1: First, we need to solve the simultaneous equations in (1) by equalling to zero in order to obtain the equilibrium solutions of prey and predator.

Step 2: Next, we reduce the nonlinear system in (1) to linear system by using Jacobian matrix. The general form for Jacobian matrix is:

$$J(x,y)=\begin{pmatrix}\frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y}\\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y}\end{pmatrix}.$$

Step 3: Now, we substitute the equilibrium solutions obtained in Step 1 into the Jacobian matrix in Step 2. Note that if we have *n* equilibrium points, then we will have *n* Jacobian matrices.

Step 4: From each Jacobian matrix in Step 3, the eigenvalues can be obtained by finding the determinant of $|\lambda I – J|=0$, where λ represents the eigenvalue and I is the identity matrix.

Step 5: Finally, the types of stability can be determined from the signs of the eigenvalues (either positive or negative). For 1-dimensional system, if λ is positive, then an equilibrium point is said to be unstable and if λ is negative, it is said to stable. For 2-dimensional system like (1), the classification of eigenvalues with the corresponding types of stability are given as follow:

a) if $0<\lambda_1<\lambda_2$, then the equilibrium point is unstable.
b) if $\lambda_1<\lambda_2<0$, then the equilibrium point is stable.
c) if $\lambda_1<0<\lambda_2$, then the equilibrium point is saddle, which implies unstable.
d) if $\lambda_1=\lambda_2>0$, then the equilibrium point is unstable.
e) if $\lambda_1=\lambda_2<0$, then the equilibrium point is stable.
f) if the eigenvalues are complex with nonzero real parts, $\lambda_1=\lambda_2 = p \pm iq$ with $p>0$, then the equilibrium is spiral unstable.
g) if the eigenvalues are complex with nonzero real parts, $\lambda_1=\lambda_2 = p \pm iq$ with $p<0$, then the equilibrium is spiral stable.
h) if the eigenvalues are complex with pure imaginary, $\lambda_1=\lambda_2 = \pm iq$, then the equilibrium is a center.

Modification of parameter of fishing rates

This method will be adapted by changing the values of parameter of fishing rates in model (2) and plot using Maple software.

RESULTS AND DISCUSSION

In this section, we analyse the proposed model in (2); the equilibria is determined, stability of equilibria is investigated and the fishing effect on both squids and snapper fish populations is varied.

Equilibria and stability of equilibria

From Step 1 in previous section, there are four possible equilibrium solutions for squids and snapper fish populations over time:

a) Extinction of both populations $(x_1,y_1)=(0,0)$,
b) Mutual exclusion where only predator survives $(x_2,y_2)=(0,9)$,
c) Mutual exclusion where only prey survives $(x_3,y_3)=(11, 0)$,
d) Coexistence of both populations $(x_4,y_4)=(10, 9)$.

Next, from Step 2, the general Jacobian matrix for model (2) is

$$J(x,y)=\begin{pmatrix} r\left(1-\frac{x+y}{K}\right)-\frac{rx}{K}-\alpha y-3Ex^2 & \frac{-rx}{K}-\alpha x\\ \frac{-sy}{K}+\alpha y & s\left(1-\frac{x+y}{K}\right)-\frac{sy}{K}+\alpha x-2Hy\end{pmatrix}.$$

From Step 3, we substitute the four equilibria obtained for model (1) into the above matrix. Therefore we will have four corresponding Jacobian matrices:

a) For (x_1,y_1), the Jacobian matrix is $J(x_1,y_1) = \begin{pmatrix} 6.5 & 0 \\ 0 & 0.5 \end{pmatrix}$,

b) For (x_2,y_2), the Jacobian matrix is $J(x_2,y_2) = \begin{pmatrix} 5.861 & 0 \\ 0.009 & -0.49 \end{pmatrix}$,

c) For (x_3,y_3), the Jacobian matrix is $J(x_3,y_3) = \begin{pmatrix} -13.08 & -0.781 \\ 0 & 0.511 \end{pmatrix}$,

d) For (x_4,y_4), the Jacobian matrix is $J(x_4,y_5) = \begin{pmatrix} -10.439 & -0.71 \\ 0.009 & -0.48 \end{pmatrix}$,

Using Step 4, we have the following corresponding eigenvalues:

a) For (x_1,y_1), the eigenvalues are $\lambda_1 = 6.5$ and $\lambda_2 = 0.5$;

b) For (x_2,y_2), the eigenvalues are $\lambda_1 = 5.861$ and $\lambda_2 = -0.49$;

c) For (x_3,y_3), the eigenvalues are $\lambda_1 = -13.08$ and $\lambda_2 = 0.511$;

d) For (x_4,y_4), the eigenvalues are $\lambda_1 = -0.481$ and $\lambda_2 = -10.438$.

Therefore, the classification of stability for each equilibrium according to Step 5 are given in Table 2.

In Table 2, the top three equilibria are unstable while the last solution is stable. The first solution (x_1,y_1) is unstable, which means that it is impossible that both populations will be extinct in the future. The instability of both (x_2,y_2) and (x_3,y_3) indicate that mutual exclusions (either only squids survives or snapper fish survives) will not occur in the future since they need each other in the marine ecosystem. Finally, the stability of the last solution (x_4,y_4) shows that the populations of prey and predator will remain in the future. However, these populations will decrease and may lead to extinction if and only if the fishing activity by the human is not controlled. The time series plot for squids and snapper fish using the values of parameters in Table 1 is shown in Figure 2. The solid line represents the squids' population size while the dashed line represents the snapper fish population size. It is noticeable that both populations are monotonically increasing and after a certain time, they stabilize to a constant values of $x = 10$ and $y = 9$, i.e. $(x_4,y_4) = (10,9)$.

Effect of fishing rates by changing the parameter

The pattern of squids and snapper fish in Figure 2 is for fixed value of fishing rate, i.e. $E = H = 0.05$. With current rate, the populations of squids and snapper fish is estimated to sustain for the next 50 years. In this section, we show the effect of fishing rate on squids and snapper fish populations by varying this rate. This analysis is fundamental to avoid both populations from extinction especially for the snapper fish since it is one of the commercial fish that has high demand in Malaysia especially in Terengganu. We plot the populations of squids and snapper fish over time for different values of fishing rates in Figure 3 and Figure 4 respectively. The fishing rates for the squids are varied for $E = 1,3,7$. When $E = 1$, the population of squids is estimated to exist and sustain for the next 50 years. Next, when the fishing rate is increased to $E = 3$, the population will decrease but still exist after 50 years. However, if $E = 7$, then the squids' population is estimated to be extinct after 50 years. This extinction occurs since $E > r$. Note that the value of squids' growth rate is $r = 6.5$ in Table 1. In an effort to guarantee that squids exist in the ecosystem, the growth rates of the squids should be greater than the fishing rates. Therefore, the fishing rates for squids must be controlled within a range of $E = (0,6.5)$.

Similar analysis is also done for the snapper fish. We vary the fishing rates for $H = 0.1,0.4,2$. When $H = 0.1$, the population of snapper fish is estimated to exist and sustain for the next 50 years. As we increase the fishing rate $H = 0.4$, eventhough the population will decrease, it will still exist after 50 years. If high fishing rate is used, for example $H = 2$, the snapper fish population is estimated to be extinct after 50 years. This extinction occurs since $H > s$. Note that the value of snapper fish growth rate is $s = 0.5$ in Table 1. In an effort to guarantee that the snapper fish exist in the ecosystem, the growth rates of the fish should be greater than the fishing rates. The range of fishing activity for snapper fish must be $H = (0,0.5)$.

Table 2. Summary of equilibria solutions for squids and snapper fish populations with their corresponding types of stability for model (2)

Equilibrium solution for squids and snapper fish population	Types of stability
$(x_1, y_1) = (0, 0)$	unstable
$(x_2, y_2) = (0, 9)$	saddle, which imply unstable
$(x_3, y_3) = (11, 0)$	saddle, which imply unstable
$(x_4, y_4) = (10, 9)$	stable

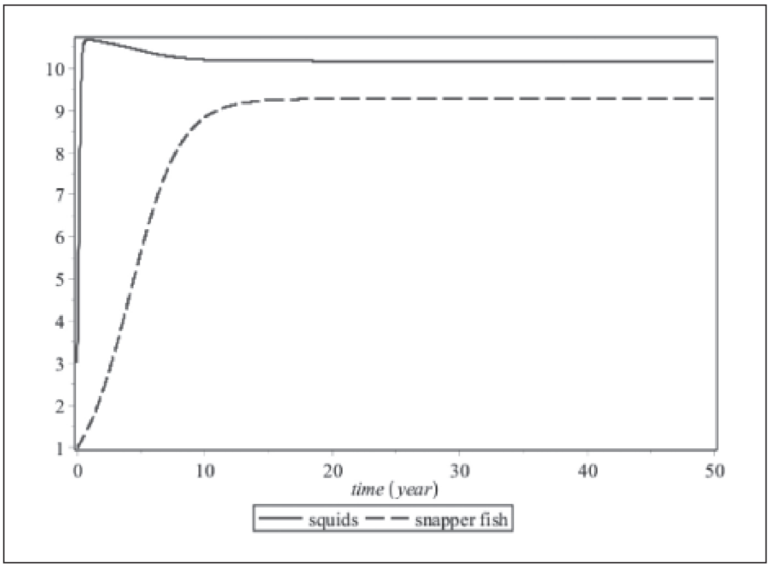

Fig. 2. Time series plot for squids and snapper fish where in the future (after 50 years), the populations will stabilize to $(x_4, y_4) = (10, 9)$ with initial population $(x_0, y_0) = (3, 1)$ for the same fishing rate $E = H = 0.05$.

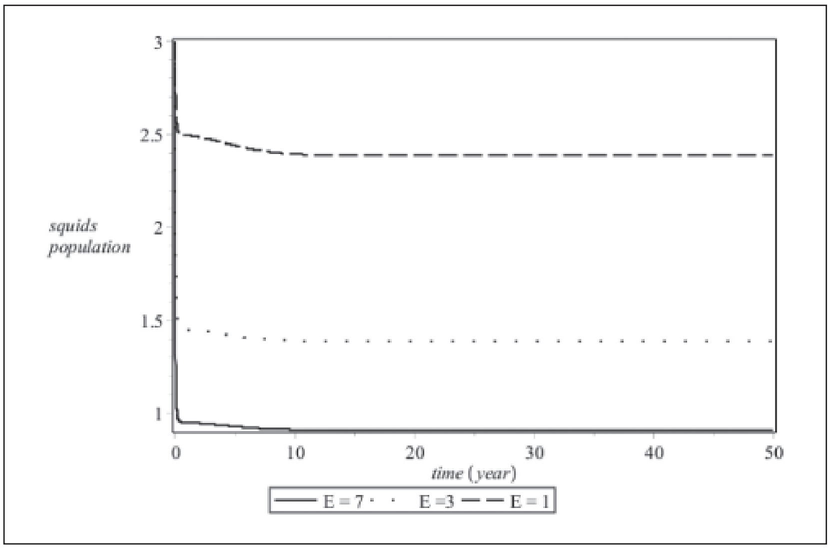

Fig. 3. The dynamics of squids population over time in year for various values of fishing rate E. The solid line shows for E = 7, the dotted line shows for E = 3 and the dashed line shows for E = 1.

In fact, the results above agree with a study by Kar and Cakraborty (2010) where they consider a prey-predator fishery model with fishing activities on the prey only. By increasing the harvesting rate on both prey and predator, both populations show changes from stable to unstable. Moreover, this fishing effort has also been investigated by Sahoo *et al.* (2016) where a bifurcation study is done by varying the fishing effort in a harvested-predator-prey model. A more recent study by Keong *et al.* (2018) shows that fishing activities give more significant impact compared to the presence of toxic substance in a prey-predator fishery model.

CONCLUSION

In this paper, a squids-snapper fish model has been developed which include both life-history characteristics and management criteria as parameters in the model. We found that this model has four equilibria for which all equilibria are unstable except for the coexistence equilibrium. This indicates that both species will remain in the future. Furthermore, by varying the fishing rates parameters, we obtained that both population would still exist in the future as long as the fishing rates can be controlled to be lower than the growth rates for both

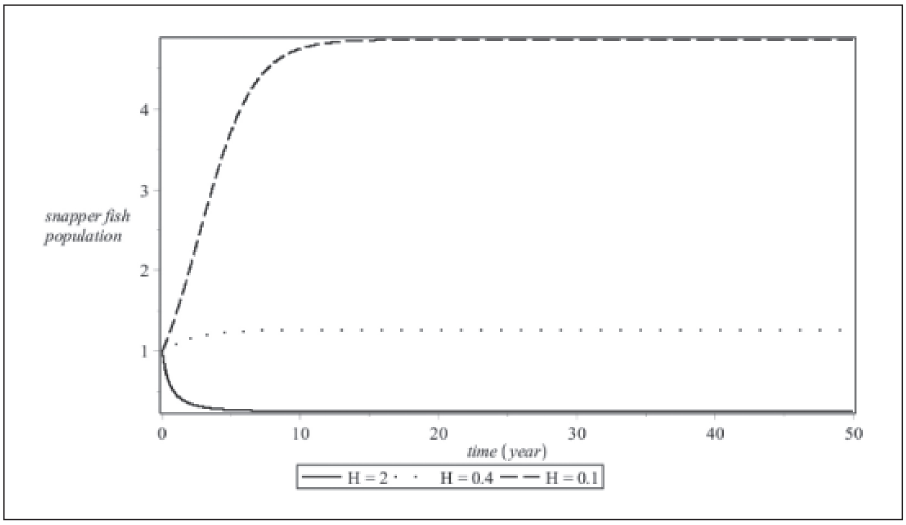

Fig. 4. The dynamics of snapper fish population over time in year for various values of fishing rate H. The solid line shows for H = 2, the dotted line shows for H = 0.4 and the dashed line shows for H = 0.1.

populations, i.e. the fishing activity for both squids and snapper fish must be within the ranges of E = (0,6.5) and H = (0,0.5) respectively. This proposed model and the obtaining results are applicable in the field of conservation on biology as well as in food security especially in Terengganu region. Our future work on the proposed model will look on the bifurcation analysis where we can determine at which value of parameter will it give the changes of stability of equilibrium point where we call this value as *bifurcation point*.

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REFERENCES

Biswas, M.H.A., Hossain, M.R. & Mondal, M.K. 2017. Mathematical modelling applied to sustainable management of marine resources. *Procedia Engineering*, **194**: 337-344.

Das, T., Mukherjee, R.N. & Chaudhuri, K.S. 2009. Harvesting a prey-predator fishery in the presence of toxicity. *Applied Mathematical Modelling*, **33**: 2282-2292.

Kar, T.K. & Chakraborty, K. 2010. Effort dynamics in a prey-predator model with harvesting. *International Journal of Information and Systems Sciences*, **6(3)**: 318-332.

Keong, A.T., Safuan, H.M. & Jacob, K. 2018a. Dynamical behaviours of prey-predator fishery model with harvesting affected by toxic substances. *MATEMATIKA*, **34(1)**: 143-151.

Keong, A.T., Safuan, H.M. & Jacob, K. 2018b. The impact of harvesting activities on prey-predator fishery model in the presence of toxin. *Journal of Science and Technology*, **10(2)**: 128-135.

Lv, Y., Yuan, R. & Yongzhen, P. 2013. A prey-predator model with harvesting for fishery resource with reserve area. *Applied Mathematical Modelling*, **37**: 3048-3062.

Lynch, S. 2014. *Dynamical Systems with Applications using Matlab*. New York: Springer.

Sahoo, B., Das, B. & Samanta, S. 2016. Dynamics of harvested-predator-prey model: role of alternative resources. *Modelling Earth Systems and Environment*, **2(140)**: 1-12.

Smith, J.M. 1974. *Models in ecology*. Cambridge University Press.

Spencer, P.D. & Collie, J.S. 1995. A simple predator-prey model of exploited marine fish populations incorporating alternative prey. *ICES Journal of Marine Science*, **53**: 615-628.

Volterra, V. 1926. Opere matematiche:mmemorie e note (Vol V). *Accademia Naz. dei Lincei*, Roma, Italy.